The gluon condensate from gauge invariant vortex vacuum texture

Kurt Langfeld

Institut für Theor. Physik, Universität Tübingen, Auf der Morgenstelle 14
D-72076 Tübingen, Germany
E-mail: langfeld@alpha8.tphys.physik.uni-tuebingen.de

In $SU(2)$ lattice gauge theory, a new self-restricted cooling procedure is developed to uncover the gauge invariant vortex vacuum texture. The emerging vortex vacuum structure amounts to the full string tension and gives rise to a mass dimension four condensate which is of pure vortex origin.

1 Introduction

A revival of the vortex picture of quark confinement arose with the construction of the p-vortices which are defined after adopting the so-called center gauge [1] by projecting the gauge fixed link variables onto center elements [1]. In this case, it was observed that the vortices are sensible degrees of freedom in the continuum limit [2]: the (area) density of the p-vortices as well as their binary interactions extrapolate to the continuum [2]. The p-vortex picture of the Yang-Mills ground state also provides an appealing explanation of the deconfinement phase transition at finite temperatures [3]. Recently, it was pointed out that the center gauge fixing which is prior to identify the physical vortex structure might be plagued by a so-called practical Gribov problem [4]. For this reason, as well as for rating the phenomenological importance of the vortices, a gauge invariant definition of the vortex vacuum texture is highly desired.

In this talk, I propose a method for obtaining gauge invariant vortex structures and investigate whether there is a genuine relation between these structures and the gluon condensate in SU(2) Yang-Mills theory. Extensive results and an adequate referencing can be found in a subsequent publication [5].

2 Vortices, gluons and vortex induced condensates

In order to exhibit the degrees of freedom responsible for confinement in SU(2) lattice Yang-Mills according to the $Z(2)$ vortex mechanism, fractionizing $SU(2)\hat{=}Z_2 \times SO(3)$ is instrumental. The corresponding "coordinates" are center vortices and coset fields. Since the $SO(3)$ coset fields are isomorphic to algebra valued fields, these degrees of freedom are identified with the *gluonic* ones.

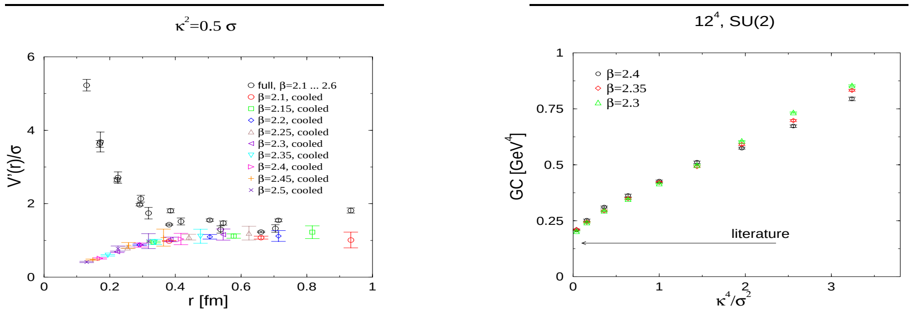

Figure 1: The static quark $Q\overline{Q}$ force as function of the distance r between quark and anti–quark for full non–Abelian and $SO(3)$ cooled configurations.

A new self–restricted cooling algorithm which reduces the $SO(3)$ action of the coset fields facilitates the gradual removal of the gluon fields from the lattice configurations while preserving the center degrees of freedom. For these purposes, I define a gluonic action density per link by

$$s^{gl}_{x,\mu} \;=\; \sum_{\bar{\nu}\neq\pm\mu} \left\{ 1 \;-\; \frac{1}{3}\,\mathrm{tr}_A\, O_{x,\mu\bar{\nu}} \right\} \;=\; \frac{1}{3} \sum_{\bar{\nu}\neq\pm\mu} F^a_{\mu\bar{\nu}}[A]\; F^a_{\mu\bar{\nu}}[A]\; a^4 + \mathcal{O}(a^6)\;, \quad (1)$$

where $O_{x,\mu\nu}$ is the plaquette calculated in terms of the $SO(3)$ part of the $SU(2)$ link elements. The sum over $\bar{\nu}$ runs from $-4\ldots4$. $F^a_{\mu\nu}[A]$ is the (continuum) field strength functional of the (continuum) gluon fields $A_\mu(x)$ and a is the lattice spacing. A local cooling step amounts for minimizing the action density (1) with respect to the coset fields at a given link. A self-restriction is imposed by rejecting the cooling of the adjoint link iff the gluonic action is smaller than some threshold value $s^{gl}_{x,\mu} \;<\; 8\kappa^4\, a^4$. Thereby κ is a gauge invariant cooling scale of mass dimension one. For $\kappa = 0$, the cooling procedure completely removes the gluon fields from the SU(2) lattice configurations leaving only center degrees of freedom. For κ smaller than the string tension, a clear signal of the vortex vacuum texture is noticed by a large SU(2) action density which is accumulated at the two dimensional vortex world sheets of the 4-dimensional space time. Since the new cooling procedure is gauge covariant and since the vortex structure manifests itself in the gauge invariant $SU(2)$ action density, the vortex texture is gauge invariant [5].

In order to study the relevance of this vortex texture, I calculated the force between a static quark anti-quark pair from $SO(3)$ cooled configurations. I find that the cooling procedure has strongly affected the force at short distances.

This is expected since the behavior at small r is dominated by the exchange of gluons, which are already partially eliminated by cooling. Most important is, however, that the string tension is unchanged by the cooling procedure.

Hadronic correlators at high momentum transfer are accessible by experiments. On the other, these correlators are sufficiently under the control of the operator product expansion (OPE), where non–perturbative properties of the Yang–Mills (or QCD) vacuum are parameterized by so–called condensates [6]. The mass dimension four operator, GC, is provided by the trace of the energy momentum tensor, and, hence proportional to the $SU(2)$ action density in the present case. Here, I will use the new cooling method to uncover the contribution of the vortices to the mass dimension four condensate of $SU(2)$ Yang–Mills theory. It turns out, that the vortex contribution to GC shows appropriate scaling towards the continuum limit [5]. The mass dimension four condensate GC in physical units is also shown in figure 1 as function of κ^4. At large values of κ, the contribution of the coset, say gluon, fields to the condensate is large and GC linearly rises with κ^4. However, the most striking feature is that GC approaches a finite value in the limit $\kappa \to 0$, thus showing a non-trivial dimension four condensate which is of pure vortex origin. The limiting value of $0.19\,\mathrm{GeV}^4$, is in rough agreement with recent values for the gluon condensate quoted in the literature [7].

Acknowledgments

It is a pleasure to thank my collaborators E.-M. Ilgenfritz and H. Reinhardt.

References

1. L. Del Debbio, M. Faber, J. Giedt, J. Greensite and S. Olejnik, Phys. Rev. **D58** (1998) 094501.
2. K. Langfeld, H. Reinhardt and O. Tennert, Phys. Lett. **B419** (1998) 317; M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, Phys. Lett. **B431** (1998) 141.
3. K. Langfeld, O. Tennert, M. Engelhardt and H. Reinhardt, Phys. Lett. **B452** (1999) 301; M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, Phys. Rev. **D 61** (2000) 054504.
4. T. G. Kovacs and E. T. Tomboulis, Phys. Lett. **B463** (1999) 104.
5. K. Langfeld, E.-M. Ilgenfritz, H. Reinhardt, hep-lat/0008017.
6. see e.g. Stephan Narison, *QCD spectral sum rules*, Singapore: World Scientific, 1989.

7. E.-M. Ilgenfritz, S. Thurner, $SU(2)$ *Field Strength Correlators: a Comparison of Cooling and RG Smoothing*, in preparation.